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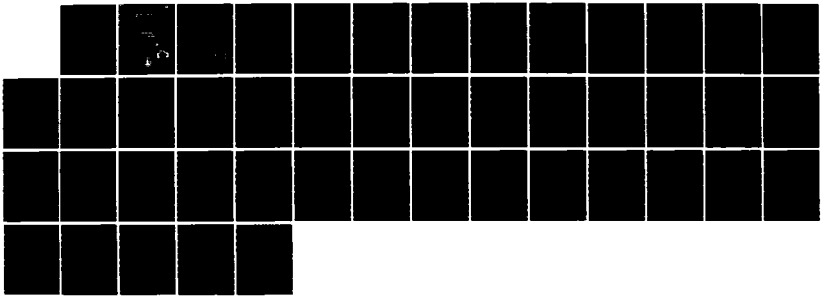

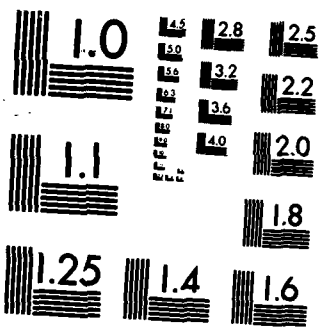

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A SURVEY OF STATISTICAL METHODS APPEARING IN THE CURRENT RUSSIAN LITERATURE ON REMOTE SENSING

BY

JOSEPH GLAZ

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1. INTRODUCTION

The articles that are being surveyed in this paper were obtained with the aid of a computerized search of two data bases: GEOREF from Dialog Information Services and COMP from BRS Information Technologies. In the GEOREF data base, there are 937 articles written by Russian scientists on various topics in remote sensing during the years 1785 through 1985. Between 1980 and 1985 alone, there are 532 articles. I have decided to search the GEOREF data base during the years 1980-1985 which yielded 159 articles. The COMP data base gave a list of 123 articles written by Russian scientists in remote sensing during the years 1976-1985. A printout of 282 article titles, their authors, and abstracts was obtained for the initial investigation.

There is no way to specify that we are interested, say, in mathematical modeling in remote sensing and obtain a complete list of the desired articles. One had to read the 282 abstracts and decide which ones seemed to be relevant to this study. Out of the 282 articles, only 63 appeared to have any statistical analysis of the observed data. After reading the 63 articles, 58 were found to be of interest. By inspecting the recent issues of the most important journals of the Russian literature on remote sensing (to be discussed below) 19 references were added, for a total of 77 articles. The most striking fact is that only 4% of the articles are not translated into English (3 out of 77 in our reference list).

There are three journals that account for approximately 90% of all the articles in our reference list. One can safely state that the research published in these three journals gives a clear picture of the remote sensing problems studies by Russian scientists. These three journals are:

1. Soviet Journal of Remote Sensing, the English translation of Issledovaniye Zemli iz Kosmosa.

2. Izvestiya, Atmospheric and Oceanic Physics, the English translation of Izvestiya, Akademii Nauk SSSR, Fizika Atmosphera Okeana.

3. Soviet Meteorology and Hydrology, the English translation of Meteorologiya i Gidrologia.

Several comments about these three journals are in order. The English translation of the first journal appears from 1980, the second journal from 1965, and the third journal from 1976. It was not clear at all from the computerized search that the Sov. J. of Remote Sensing is indeed the English translation of Issledovaniye Zemli iz Kosmosa, as the first name appeared in GEOREF and the second in COMP. Moreover, it was not stated that Izvestiya, Akademii Nauk SSSR, Fizika Atmosphera Okeana has been translated into English, and all the references were given in the data base for the original articles in Russian.

We now proceed to give a brief description of the organization of this review article. In the second section of this article, we will review the major areas of remote sensing and state the statistical techniques that were used in each of the areas. In Section 3, we will discuss briefly each of the statistical techniques that have been mentioned in Section 2. Then, we will conclude our survey with a short summary followed by a bibliography.

2. AREAS OF APPLICATION OF REMOTE SENSING TECHNIQUES

The articles included in this study can be classified into four major areas: Agriculture, Geology, Meteorology, and Oceanography. The percentage breakdown of the total number of articles in each area is: 21% (16 out of 77), 21% (16 our of 77), 39% (30 out of 77), and 19% (15 out of 77), respectively.

We now proceed to describe the types of problems studied in each of the areas mentioned above and list the statistical techniques that have been used to analyze the observed data.

a. Agriculture. The following topics are discussed in this area of application of remote sensing techniques: evaluating the status of farmland by monitoring the optical parameters of the atmosphere ([2], [11], [43]), estimation and identification of the states of vegetative cover from airborne observations ([12], [39]), estimating biomass of grass and semishrub vegetation ([75]), evaluation of the accuracy of land class determination and forest sites ([29]), interpretation of aerospace photographs of forests ([4], [23]), analysis of canopy-soil optical reflectance ([30], [54], [77]), and radiation studies of soils and vegetative covers ([5], [40], [67]).

We now proceed to list the statistical techniques employed in these articles: point estimation of means and standard deviations ([2], [5], [12]), classification analysis ([4], [29]), discriminant analysis ([23], [29]), cluster analysis ([73]), analysis of variance ([30]), regression analysis ([5], [11], [12], [39], [75]), stochastic modeling ([40], [43], [54], [67], [75], [77]), simulation ([40], [67], [77]), spatial analysis ([77]), and spectral analysis ([5]).

b. Geology. The following topics are discussed here: earth resources surveying ([6], [52], [56], [66], [68]), choosing the spectral zones to distinguish the natural objects on the earth surface by their spectral characteristics ([9]), determining the state of ground objects ([10]), earth exploration by using optical imaging systems ([17], [27], [70], [72]),

geological investigation from observations made from space ([20], [42], [71]), developing and controlling an earth sensing space system ([53]), and formulating a signal-to-noise ratio concept for a spaceborne photographic system ([49]).

We now list the statistical techniques used in these articles: classification analysis ([10], [27], [42], [52], [71]), discrim nant analysis ([27]), econometrics ([6], [53]), data validation ([52), filtering theory ([20]), information theory ([49]), modified MLE ([10]), nonlinear estimation ([10]), optimal sampling ([9], [70]), optimal decision control ([53]), optimal linear prediction ([56]), stochastic modeling ([6], [17], [66], [68]), spatial analysis ([20], [72]), and spectral analysis ([72]).

c. Meteorology. The following topics are discussed in this area of application of remote sensing techniques: statistical characteristics of the spatial thickness of the atmosphere ([1]), determining integral moisture content in a cloudless atmosphere ([3], [47]), temperature-wind atmosphere sounding by radioacoustic methods ([7], [15], [21]), scattering of radar signals by clouds and percipitation ([8]), examining the informativeness of aerological and remote sensing systems ([13], [14], [28], [51]), mathematical models for cloud cover ([16], [50]), interpreting the characteristics of the outgoing thermal radiation field of earth-atmosphere system ([7], [58], [59], [60], [61], [62], [63], [65]) construction of snow cover maps from data provided by meteorological satelites ([25]), estimating the volume coefficient of absorption by aerosols from airborne measurements of spectral fluxes ([31]), determining the earth's surface

temperatures and weather forecasting ([32], [57], [64], [74]), automated analysis of aerospace photographs in monitoring polution ([38]), dependence of signal processing on optical parameters ([41]), laser beam intensity fluctuations spectra in atmospheric precipitation ([76]).

The statistical techniques that have been employed to analyze the data in articles mentioned above are: point estimation of means and standard deviations ([7], [8], [25], [31], [41]), classification analysis ([38], [51]), cluster analysis ([16], [38]), inverse problems in linear and nonlinear models ([47], [58], [60], [61], [63], [64]), multivariate estimation ([14], [51], [63], [64]), principal component analysis ([1], [28]), regression models ([3], [32], [47], [50], [57], [58], [62], [63], [65]), sampling techniques ([13]), simulation ([16], [41], [47]), spatial analysis ([22], [61], [62], [64], [76]), spectral analysis ([57], [76]), filter theory ([22], [59], [63]), stochastic modeling ([15], [16], [21], [50], [59]).

d. Oceanography. The following topics are studied in this area: radiometric discrimination of sea ice ([18]), remote radiometric methods of investigating sea surfaces ([19], [35], [37]), interpretation of remotely sensed ice cover data from its thermal emission ([24]), acoustic tomography of the ocean ([26]), analysis of remotely sensed data of internal waves ([33], [44]), spatial structure of the radiance field for the ocean-atmosphere system ([34]), analysis of the structure of sea surface temperature field ([36], [45], [46]), year-to-year variation of the radiation budget of ocean surface ([48]), optimal procedures for determining the sea surface and atmospheric parameters ([55], [69]).

The statistical methods discussed in these articles are: point estimation of means and standard deviations ([19], [44]), classification analysis ([18]), discriminant analysis ([37]), regression models ([37], [45], [46], [55], [69]), optimal design ([26]), spatial analysis ([34], [35], [36], [45], [46], [48]), spectral analysis ([33], [34], [48]), stochastic modeling ([24], [36], [37], [46], [69]).

3. STATISTICAL TECHNIQUES USED IN REMOTE SENSING

In this section we briefly describe some of the statistical techniques that have been used in the current Russian literature on remote sensing.

a. Estimation ([2], [5], [7], [8], [10], [12], [14], [19], [25], [31], [41], [44], [51], [74]).

The references cited above do not include articles that discuss estimation for regression or spatial models. Most of the estimation techniques used here are quite elementary, though in some cases the mathematical models that describe the physical phenomena can be quite involved and far beyond the scope of knowledge of a statistician.

In [2] the authors estimate the var(J), where $J = I_1/I_2$ and I_1 and I_2 are the radiances at wavelengths λ_1 and λ_2, respectively. Optical parameters are related to radiance by the following equation: $I = I_0 \exp(-\tau)$, where τ is the optical thickness of the atmosphere. For the two wavelengths λ_1 and λ_2 the variance covariance matrix for τ_1 and τ_2 is assumed to be $\begin{pmatrix} \sigma_{11}^2 & \sigma_{12}^2 \\ \sigma_{12}^2 & \sigma_{22}^2 \end{pmatrix}$ and $\sigma_{12}^2 = \sigma_{11}\,\sigma_{22}\,R_{12}$, where R_{12} is the correlation coefficient. The authors use the fact $J = J(\tau_1,\tau_2) = e^{-(\tau_1-\tau_2)}$ and use the first term of

the Taylor series expansion for $J(\tau_1,\tau_2)$ evaluated at $(E(\tau_1), E(\tau_2))$ to obtain an approximation:

$$\text{var}(J) \simeq e^{-2[E(\tau_1)-E(\tau_2)]}(\sigma_{11}^2 + \sigma_{22}^2 + 2\sigma_{11}\sigma_{22}R_{12}) .$$

For several cities in Russia E(J) and the approximation for var(J) are estimated. The quantity $\hat{\varepsilon} = \hat{\sigma}_J/\widehat{E(J)}$ is used as an estimate for the maximum relative error in their estimation procedure. In the paper itself the authors define $\varepsilon = \sigma_J/J$.

In [10] a uniform surface H is partitioned into a large number of elements, N, called macroelements. Each of these N elements is divided into elementary cells, called microelements, which characterize the specific properties of an object in H. Without loss of generality it is assumed that the number of microelements is constant and equal to its mean value $n >> 1$. The state of each microelement is described by the vector $\underset{\sim}{x}$ with pdf $P(\underset{\sim}{x})$. The radiance of a microelement is described by $\underset{\sim}{y}$ and it is assumed that $\underset{\sim}{y} = f(\underset{\sim}{x})$ for some function f. In this paper, the authors treat $\underset{\sim}{x}$ and $\underset{\sim}{y}$ as scalars.

Now, if y_i and x_i are the radiance and state variables, respectively, of an ith microelement in a macroelement, then $Y = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $X = \frac{1}{n}\sum_{i=1}^{n} x_i$ are the mean radiance and state vector of an object for an arbitrary macroelement. The stochastic model for Y as a function of X is discussed via the regression curve of Y on X and the residual variance after regression.

The central problem that concerns the authors in this paper is to estimate the moments of X (most often the first two) given the data $Y_1, Y_2, \ldots, Y_N$.

They use the maximum likelihood approach and consider the following log-likelihood equation:

$$\sum_{i=1}^{N} \nu \log P(Y_i, a_1, \ldots, a_L, \mu_1, \ldots, \mu_{M^*}) / \nu \mu_k = 0 \ , \quad k = 1,2,\ldots,M < M^* \ ,$$

where a_q are the coefficients in the series used to approximate the function f(x):

$$f(x) = \sum_{q=0}^{L-1} a_q x^q \ .$$

P(Y) is substituted into the log-likelihood equation as an Edgeworth series with M terms. Hence only M out of M* parameters can be estimated and $\mu_{M+1}, \ldots, \mu_{M^*}$ are the nuisance parameters. In this article the nuisance parameters are expressed as functions of $\mu_1, \ldots, \mu_M$. These functions are chosen to maximize the entropy of the P(x) distribution:

$$S = - \int_{-\infty}^{\infty} P(x) \log P(x) dx.$$

To study the precision of the estimates obtained using this approach the authors suggest evaluating the variance for the μ_k estimate due to μ_ℓ moment fluctuation:

$$\sigma^2_{\mu_k} = c \int_{-\infty}^{\infty} e^{S(\mu_\ell)} (\mu_\ell - \mu_\ell^0)^2 (\nu \mu_k / \nu \mu_\ell)^2 d\mu_\ell$$

where k = 1,...,M, ℓ = M+1,...,M*, μ_ℓ^0 is the fixed value assigned to μ_ℓ, c is a normalizing constant and ν denotes the partial derivative.

At the end of the article the authors consider distributions from the exponential class as an approximation for P(Y):

$$\exp[-\frac{\tilde{Y}^2}{2}] \ \exp[\sum_{j=3}^{M} n^{1-j/2} \ T_j(\tilde{Y})] ,$$

where $\tilde{Y} = \frac{Y - \xi_1}{\sqrt{\xi_2}}$ is a standardized value of Y. They use the Cramer-Rao inequality to estimate the minimal variance of the estimates for $\mu_1,\ldots,\mu_M$, obtained by the modified maximum likelihood procedure discussed above.

In [41] Monte Carlo techniques were used to study the degree of polarization of the multiple scattered background reflected by clouds. The dependence of total signal $\delta_b(\tau)$ on the optical sensing depth τ and the characteristic parameter η, which determines the optico-geometric illumination conditions, had been studied. The depolarization of second-scattering signal was estimated as a function of τ and η. Standard error for this estimate was computed.

The authors conclude that at an optical depth $\tau \simeq 1$ and $\tau \leq .02$, the double scattering signal represent about 80% of the entire background for both cloud models considered. As τ increases as well as the penetration depth of the radiation into the cloud, the fraction of the double scattering signal in the relected background drops off significantly.

In [44] the authors investigate how the internal wave modifies the structure of the surface wave motion and how and under what conditions these changes will influence the image of the ocean surface.

Let the image of solar glitter at the ocean surface form in a receiving device located at height h above the sea level. The receiving device consists of a large number of elementary receivers with narrow directivity patterns, forming together the viewing field of the instrument. Under various assumptions the authors state that a random occurrence of

power onto an elementary receiver is given by

$$(1) \qquad P = \frac{E_0\Omega^2}{\pi(\omega^2+\Omega^2)} \iint_{-\infty}^{\infty} \exp\left\{ - \frac{(\underset{\sim}{r}-\underset{\sim}{r}_0)^2}{\beta^2 h^2} - \frac{(\underset{\sim}{r}_0-\underset{\sim}{r}+3\underset{\sim}{\eta}h-\underset{\sim}{x}_{1\perp}-\underset{\sim}{x}_{2\perp})^2}{h^2(\omega^2+\Omega^2)} \right\} d\underset{\sim}{r} ,$$

where E_0 is the illumination of the marine surface by direct solar rays, ω and Ω are the angular dimensions of the solar disk and of the receiving aperture from the surface, β is the width of the directivity pattern, $\underset{\sim}{x}_1$ and $\underset{\sim}{x}_2$ are unit vectors characterizing the direction of the solar rays and the receiving pattern, $\underset{\sim}{x}_{1\perp}$ and $\underset{\sim}{x}_{2\perp}$ are their projections on the horizontal plane, $\underset{\sim}{r}_0$ is a vector describing the orientation of the receiver in this plane, and $\underset{\sim}{\eta}$ is the projection of the local normal to the surface onto the plane. Averaging (1) over the observed data on the surface and assuming that its slopes have a normal distribution and that the wave motion is isotropic, the authors have obtained

$$(2) \qquad E(P) = \frac{E_0\Omega^2h^2\beta^2}{8\sigma_\theta^2} \exp\left\{ - \frac{(\underset{\sim}{x}_{1\perp}+\underset{\sim}{x}_{2\perp})^2}{8\sigma_\theta^2} \right\} ,$$

where σ_θ^2 is the variance of the surface slopes. Moreover, the relationship between the change in the mean power onto an elementary receiver and the change in the variance of surface slopes is obtained:

$$(3) \qquad E(\Delta P) = -E(P)\Delta\sigma_\theta^2/\sigma_\theta^2 .$$

Based on that, the minimum recordable level of wave disturbance at which the internal wave can induce detectable changes in the surface images is estimated. This analysis is repeated for another method of recording the sea state, in the case when the radiation incident onto the receiver has been scattered by the atmosphere.

In [74] the geopotential field in the "true" state at time $t = t_j$ is denoted by $\underset{\sim}{H}_j = \underset{\sim}{H}(t_j)$ and the observed geopotential values by $\underset{\sim}{X}_j = \underset{\sim}{X}(t_j)$. The observed data consists of $\underset{\sim}{X}_0, \underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_k$ at times $t_0 < t_1 < \ldots < t_k$. The authors are interested in predicting $\underset{\sim}{H}_k$.

Assuming that $P(\underset{\sim}{X}_k | \underset{\sim}{H}_k) = P(\underset{\sim}{X}_k | \underset{\sim}{H}_k, \underset{\sim}{X}_0, \ldots, \underset{\sim}{X}_{k-1})$, the authors obtain the following representation:

$$(1) \qquad P(\underset{\sim}{H}_k | \underset{\sim}{X}_0, \ldots, \underset{\sim}{X}_k) = \text{const. } P(\underset{\sim}{H}_k | \underset{\sim}{X}_0, \ldots, \underset{\sim}{X}_{k-1}) P(\underset{\sim}{X}_k | \underset{\sim}{H}_k).$$

They use (1) to estimate the true state of $\underset{\sim}{H}$ at time $t = t_k$ from $\underset{\sim}{X}_0, \underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_k$, by choosing $\underset{\sim}{H}_k$ that maximizes (1). To evaluate $P(\underset{\sim}{X}_k | \underset{\sim}{H}_k)$, they assume that $\underset{\sim}{X}_j = B_j \underset{\sim}{H}_j + \underset{\sim}{N}_j$, where B_j is the time-variable conversion matrix and $\underset{\sim}{N}_j$ is the random error independent from $\underset{\sim}{H}_i$'s. $E(\underset{\sim}{N}_j) = 0$ and $E(\underset{\sim}{N}_i \underset{\sim}{N}_j^T) = C_N \delta(t_i - t_j)$ is the covariance matrix. Then

$$(2) \qquad P(\underset{\sim}{X}_k | \underset{\sim}{H}_k) = \text{const. } \exp\{-\tfrac{1}{2}(\underset{\sim}{X}_k - B_k \underset{\sim}{H}_k)^T C_N^{-1} (\underset{\sim}{X}_k - B_k \underset{\sim}{H}_k)\}.$$

To derive the formula for $P(\underset{\sim}{H}_k | \underset{\sim}{X}_0, \ldots, \underset{\sim}{X}_{k-1})$, the authors assume that the geopotential information for times $t < t_k$ can be accumulated in the geopotential forecast $\underset{\sim}{H}_k^f$ calculated for time $t = t_k$ from $\underset{\sim}{X}_0, \underset{\sim}{X}_1, \ldots, \underset{\sim}{X}_{k-1}$. Hence, we use the following approximation:

$$P(\underset{\sim}{H}_k | \underset{\sim}{X}_0, \ldots, \underset{\sim}{X}_{k-1}) \simeq P(\underset{\sim}{H}_k | \underset{\sim}{H}_k^f)$$

and approximate it by a multivariate normal distribution:

$$(3) \qquad P(\underset{\sim}{H}_k | \underset{\sim}{X}_0, \ldots, \underset{\sim}{X}_{k-1}) \simeq \text{const. } \exp\{-\tfrac{1}{2}(\underset{\sim}{H}_k - \underset{\sim}{H}_k^f)^T (C_k^f)^{-1} (\underset{\sim}{H}_k - \underset{\sim}{H}_k^f)\},$$

where C_k^f is the forecasting error covariance matrix. Substituting (2) and (3) in (1) yields:

$$(4) \qquad P(\underset{\sim}{H}_k | \underset{\sim}{X}_0, \ldots, \underset{\sim}{X}_k) = \text{const. } \exp\{-\tfrac{1}{2}(\underset{\sim}{H}_k - \underset{\sim}{H}_k^f)^T (C_k^f)^{-1} (\underset{\sim}{H}_k - \underset{\sim}{H}_k^f) - \tfrac{1}{2}(\underset{\sim}{X}_k - B_k \underset{\sim}{H}_k)^T C_N^{-1} (\underset{\sim}{X}_k - B_k \underset{\sim}{H}_k)\}.$$

$\underset{\sim}{H}_k$ is estimated by maximizing (4) (provided we know C_k^f and C_N):

$$\hat{\underset{\sim}{H}}_k = \underset{\sim}{H}_k^f + K_k(\underset{\sim}{X}_k - B_k\underset{\sim}{H}_k^f)$$

where

$$K_k = C_k^f B_k^T (B_k C_k^f B_k^T + C_N)^{-1}.$$

The authors discuss how to estimate C_k^f and C_N.

b. Classification and Discrimination ([4], [10], [18], [23], [27], [29], [37], [38], [42], [51], [52], [71]).

In this paper, [18], a Bayesian classification rule is used for radiometric discrimination of sea ice types.

Let $\underset{\sim}{x} = (x_1,\ldots,x_n)$ be the vector of characteristics of a given image. $P(\omega_i|\underset{\sim}{x})$ denotes the conditional probability of the image to be classified in class ω_i, given it corresponds to the observed characteristics $\underset{\sim}{x}$, $i = 1,2,\ldots,M$. If L_{ij} is the loss that one suffers if an image that belongs to class j was classified into class i, then the mean value of losses due to misclassification is given by

$$\tau_j(\underset{\sim}{x}) = \sum_{i=1}^{M} L_{ij}P(\omega_i|\underset{\sim}{x}) = \frac{\sum_{i=1}^{M} L_{ij}P(\underset{\sim}{x}|\omega_i)P(\omega_i)}{\sum_{k=1}^{M} P(\underset{\sim}{x}|\omega_k)P(\omega_k)},$$

where $P(\omega_i)$ are a priori probabilities that the object chosen at random belongs to class ω_i. The classification rule that minimizes the total losses assigns the image $\underset{\sim}{x}$ to class ω_i if

$$\tau_i(\underset{\sim}{x}) < \tau_j(\underset{\sim}{x}), \quad i \neq j,\ j = 1,\ldots,M.$$

Under the assumption of 0-1 loss, the above translates to

$$P(\underset{\sim}{x}|\omega_i)P(\omega_i) > P(\underset{\sim}{x}|\omega_j)P(\omega_j).$$

Experiments on radio thermal radiation of sea ice of different types show that the mean values of brightness temperatures at two wavelengths (1.6 and 3.2 cm.) can be used as characteristics. Empirical probability distribution for brightness temperatures were constructed for each of the ice types and wavelengths. These were used to approximate $P(\underset{\sim}{x}|\omega_j)$. The $P(\omega_i|\underset{\sim}{x})$ were then evaluated and used in the classification procedure. To improve the radiometric discrimination of ice types, the authors found it is necessary to use a digital nonlinear lowpass filter, which smooths only those values of brightness temperature which differ from the current mean value with not more than a specified value ε. They also come to a conclusion that using infrared channel data makes it possible to distinguish thin ice.

In [29] the authors are proposing a procedure to evaluate a classification rule in forestry studies.

Assume that there are N classes of features to be recognized in a given image. Define for each ith class n_i fixed test sites of equal area and group them into two sequences: a reference sequence of n_{ir} sites and a control sequence of n_{ic} sites. The reference sample will be used to define the classification procedure and the control sample to evaluate its accuracy. For the control sample, each observation receives three scores: correct classification, misclassification or omission (the computer cannot decide).

Suppose that of all n_{ic} sites from the ith class presented to the computer n_{p_i} were classified correctly, n_{q_i} misclassified and n_{r_i} omitted,

with $n_{p_i} + n_{q_i} + n_{r_i} = n_{ic}$. Define $p_i = (n_{p_i}/n_{ic})100$ as percent of correct classificationsm $q_i = (n_{q_i}/n_{ic})100$ as percent misclassifications and $r_i = (n_{r_i}/n_{ic})100$ as percent of omissions. Relative misclassification of class i with respect to all $j \neq i$ classes is defined as the sum of misclassification rates

$$q_i = \sum_{j=1}^{N} q_{ij} ,$$

where q_{ij} is percent data from class i that was misclassified into class j. The authors define a decision matrix, denoted by n, that gives the computer recognition decisions for all the sites:

$$n = \begin{bmatrix} n_{p1} & n_{q12} & \cdots & n_{q1N} & n_{r1} \\ n_{q21} & n_{p2} & \cdots & n_{q2N} & n_{r2} \\ \vdots & \vdots & & \vdots & \vdots \\ n_{qN1} & n_{qN2} & & n_{pN} & n_{rN} \end{bmatrix}_{Nx(N-1)}$$

Define $n_p = \sum_{j=1}^{N} n_{pj}$, the total number of correct classifications and $n_c = \sum_{j=1}^{N} n_{jc}$ the total number of control sites. Then $p = (n_p/n_c)100$ is the average percentage of correct classifications in a given test. By adding the decision matrix the sum row containing n_c, n_p and p, we obtain what the authors call an "extended decision matrix."

This extended decision matrix is used to compare two classification techniques that are based on space scanner image processing: single and two-step color selection. The single step color classification method interprets in the interactive mode multiband and false-color forest images

with the aim of identifying various forest areas differing by the dominant species. With this method the optical densities of registered band image elements provide color signatures for the features to be recognized.

Let $\underset{\sim}{x} = (x_1,\ldots,x_N)$ be the N-dimensional vector of color signatures where N is the number of band images. The classification rule used is the same as in [18] for the 0-1 loss function. The discriminant function for the kth and jth class is given by

$$F_{kj} = P(\omega_k)P(\underset{\sim}{x}|\omega_k) - P(\omega_j)P(\underset{\sim}{x}|\omega_j)$$

which is positive for $\underset{\sim}{x}$'s that lead to a classification into the kth class. Since $P(\underset{\sim}{x}|\omega_k)$ is usually unknown, the following nonparametric estimate is used:

$$P(\underset{\sim}{x}|\omega_k) \approx \frac{1}{M} \sum_{i=1}^{M} \frac{1}{V_M} v\left(\frac{\underset{\sim}{x} - \underset{\sim}{x}_{ki}}{h}\right)$$

where M is the length of the reference sample, V_M is the volume of the elementary hyperwindow,

$$v\left(\frac{\underset{\sim}{x} - \underset{\sim}{x}_{ki}}{h}\right) = \begin{cases} 1 & \text{for } \dfrac{|\underset{\sim}{x} - \underset{\sim}{x}_{ki}|}{h} \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases},$$

and h is the window side. Hence

$$F_{kj}(x) = \frac{P(\omega_k)}{M} \sum_{i=1}^{M} \frac{1}{V_M} v\left(\frac{\underset{\sim}{x}-\underset{\sim}{x}_{ki}}{h}\right) - \frac{P(\omega_j)}{M} \sum_{i=1}^{M} \frac{1}{V_M} v\left(\frac{\underset{\sim}{x}-\underset{\sim}{x}_{ki}}{h}\right) .$$

The size h is selected to match the length of the sample: it decreases for larger M. With integer valued signature $h_{min} = 1$ and hence

$$F_{kj} = P(\omega_k) \sum_{i=1}^{M} v(\underset{\sim}{x} - \underset{\sim}{x}_{ki}) - P(\omega_j) \sum_{i=1}^{M} v(\underset{\sim}{x} - \underset{\sim}{x}_{ji}) ,$$

i.e., it is a weighted difference of the corresponding empirical distributions.

The two-step color selection is an extension of the previous technique and will not be discussed here in detail. The comparison of the decision matrices for the two procedures show that the second one is slightly better. The decision matrix is also used to study how the size of reference samples affects correct classification.

In [52] the authors are interested in the problem of estimating the validity of remotely sensed data coming from several sources and generating contradictory messages. It is assumed that the data sources are independent. The statistical problem can be formulated as follows: the object under study belongs to one of the M states, ω_j = j = 1,...,M, with a priori probabilities $P(\omega_j)$, j = 1,...,M. Assume that the data processing subsystem (DPS) receives information about the subject under study from n independent sources. Based on that, we get a message ν_j, the object is in state ω_j, j = 1,...,M. Let $\underset{\sim}{x}_i$ be the vector of observations representing the ith source, i = 1,...,n. One component of the state $\underset{\sim}{x}_i$ is a random error vector $\underset{\sim}{\varepsilon}$. Let $P^i_{kj}(\underset{\sim}{x}_i,t)$ be the conditional probability of the ν_j message coming from the ith source and the event ω_k. Assume that these conditional probabilities are known functions of $\underset{\sim}{x}_i$ and t. $Q_i(\underset{\sim}{x},t)$ is the information matrix of the ith source: $Q_i(\underset{\sim}{x},t) = [P^i_{kj}(\underset{\sim}{x}_i,t)]_{M\times M}$, i = 1,...,n. A decision, based on the above probabilities, is taken to generate a particular message. The authors want to evaluate the probability of getting a correct message (classification).

Assume that P^i_{kj} are known sampled values of the respective $P^i_{kj}(\underset{\sim}{x}_i,t)$. Each observed result is associated with an n-dimensional message vector $\underset{\sim}{S}$, such that its ith coordinate is equal to j if the ith source generated the message ν_j. There are $N = M^n$ different vectors $\underset{\sim}{S}_q$, q = 1,...,N. With each $\underset{\sim}{S}_q$ the authors associate the Bayes a posteriori probabilities

$$P(\omega_k|\underset{\sim}{S}_q) = \frac{P(\omega_k)P(\underset{\sim}{S}_q|\omega_k)}{\sum_{k=1}^{M} P(\omega_k)P(\underset{\sim}{S}_q|\omega_k)} .$$

Since the data sources are independent, then

$$P(\underset{\sim}{S}_q|\omega_k) = \prod_{\substack{i=1 \\ j = S^i_q}}^{n} P^i_{kj} ,$$

where S^i_q is the ith coordinate of $\underset{\sim}{S}_q$. Let $\tilde{\nu}_j$, j = 1,...,M, be the messages at the DPS output, meaning the object is in state ω_j. Subject to several assumptions, the authors derive the probability that the generated message at the DPS output is $\tilde{\nu}_j$ when, in fact, ω_k is the true state:

$$P(\tilde{\nu}_j|\omega_k) = \sum_{q=1}^{N} P(\tilde{\nu}_j|\underset{\sim}{S}_q)P(\underset{\sim}{S}_q|\omega_k).$$

The DPS information matrix Q is defined as $Q = [P(\tilde{\nu}_j|\omega_k)]_{M\times M}$. The diagonal elements of Q, $P(\tilde{\nu}_j|\omega_j)$, are the conditional probabilities of correct classifications. $W(\underset{\sim}{p})$ denotes the data validity as a function of $p_j = P(\omega_j)$ is defined:

$$W(\underset{\sim}{p}) = \sum_{j=1}^{M} p_j \, P(\tilde{\nu}_j|\omega_j).$$

Since $P(\tilde{\nu}_j|\omega_j)$ are determined by the entries Q_i via the formulas for $P(\nu_k|\underset{\sim}{S}_q)$ and $P(\underset{\sim}{S}_q|\omega_k)$, W is a function of $M^2 \times n + M$ variables. The authors

investigate $W(\underset{\sim}{p})$ for the case M = 2 and n = 2 for various matrices Q_i. They observe that $W(\underset{\sim}{p})$ is minimum for $\underset{\sim}{p} = (\frac{1}{M},\ldots,\frac{1}{M})$ and that it varies slowly about this minimum value if n > 2. They suggest using this lower bound as an approximation for the data validity.

c. Cluster Analysis ([16], [38], [73]).

In [73] a technique is presented for clustering farmlands on the basis of spectral radiance measurements using the relative spatial position of fields as an auxiliary variable. The following assumption is made: two adjacent fields occupied by cultivated vegetation belong to different classes.

Assume that n measurements of spectral radiance have to be clustered. Internal structure links are introduced so that the set of measurements are ordered. This representation of the measurements creates a graph $G = (V,E)$, where $V = \{O_1,O_2,\ldots,O_n\}$ is the set of vertices and $E = \{\ell_1,\ell_2,\ldots,\ell_m\}$ is the set of edges. If the set E is ordered as $\ell_1 \leq \ell_2 \leq \ldots \leq \ell_m$, then G is an ordered graph. $O_i \in V$ are referred to as objects and $\ell_k \in E$ are the links. A proximity graph $P = (V,E)$ is an ordered graph where V is the set of objects to be clustered and E is the set of links. The links $\ell_1,\ell_2,\ldots,\ell_m$ may have different meanings:

1. Ruling out the inclusion of points of an edge in the same cluster.
2. Allowing the objects of an edge to be in the same cluster but requiring the ordering relation of the links to be examined.

If $\ell_i = O_pO_q$ and $\ell_j = O_rO_s$, then $\ell_i \leq \ell_j$ implies that the degree of similarity between O_p and O_q is at least the same as between O_r and O_s.

The authors have used this graph theoretic clustering procedure for a farmland inventory from multispectral space and air photos. A territory of the Wroclav province in Poland with 213 farms was used in this study. The average values of optical densities were determined for each field. A proximity graph was then constructed from these densities. Based on this technique the 213 fields were clustered in 13 clusters. The authors have found this technique to be very satisfactory in solving practical crop inventory problems.

d. Sampling ([9], [13], [70]).

This paper, [9], discusses a procedure for selecting spectral channels for collecting remote sensing data on the basis of Kullback information measure.

Let $\underset{\sim}{x}$ be an observed vector of spectral brightness coefficient in n different channels. Based on these measurements various objects have to be discriminated. For the sake of simplicity the authors assume that there are two classes of objects, whose features are described by pdf's $f_1(\underset{\sim}{x})$ and $f_2(\underset{\sim}{x})$. The Kullback information measure for difference between $f_1(\underset{\sim}{x})$ and $f_2(\underset{\sim}{x})$ is defined

$$J(1;2,\underset{\sim}{x}) = \int \{f_1(\underset{\sim}{x}) - f_2(\underset{\sim}{x})\} \ln \frac{f_1(\underset{\sim}{x})}{f_2(\underset{\sim}{x})} \, d\underset{\sim}{x} .$$

The authors assume that $f_i(\underset{\sim}{x})$ has a multivariate normal distribution with mean $\underset{\sim}{\mu}_i$ and covariance matrix Σ_i, i = 1,2. They evaluate the increase in $J(1;2,\underset{\sim}{x})$ when n is increased to n + 1, denoted by $\Delta J(1;2,\underset{\sim}{x})$. The authors proceed to investigate the conditions under which $\Delta J(1;2,\underset{\sim}{x}) = 0$. Assuming

$\rho^{i}_{n,n+1} = \mathrm{cov}(X^{i}_{n}, X^{i}_{n+1}) \simeq 1$ (in fact, they could have assumed that $\mathrm{cov}(\sum_{i=1}^{n} a_i X_i, X_{n+1}) \simeq 1$), in order to evaluate $\Delta J(1;2,x)$ one has to perturb the $n+1$ dimensional covariance matrices by λI, where $\lambda > 0$ and I is a $(n+1) \times (n+1)$ identity matrix and at the end let $\lambda \to 0$. The condition of $\Delta J(1;2,\underset{\sim}{x}) = 0$ at $\lambda \to 0$ is that

$$\gamma^1 = \gamma^2 = \frac{r_{n+1}}{r_n} \tag{1}$$

where $\gamma^i = \dfrac{\sigma^i_{n+1,n+1}}{\sigma^i_{n,n}}$ is the ratio of standard deviations for X_{n+1} and X_n with respect to f^i and $r_j = E(X^1_j) - E(X^2_j)$. Relation (1) makes it possible to select the spectral channels that contain new information. A threshold is defined beyond which the addition of channels does not provide any significant information.

e. <u>Stochastic Modeling and Simulation</u> ([6], [15], [16], [17], [21], [24], [36], [37], [40], [41], [43], [46], [47], [50], [54], [59], [66], [67], [68], [69], [77]).

In [16] the author is interested in modeling spatial and time variations in the distribution of cloud cover over Earth's surface. The authors state that for distances r less than 3-5 thousand kilometers and time intervals τ less than 4-6 days, the successive observations are correlated. In satellite-based earth observations, the distribution of cloud cover over the earth's surface may be defined as a discrete space-time random field $U(\lambda,\psi,t)$, which takes on values: 0 if there is no cloud cover over the observation point with geographical coordinate λ, ψ and time t, and 1 if cloud cover

is present over that observation point. It is assumed that the cloudiness process B(t) over a certain averaging region, S_A, has integer values 0,1,...,10. It is determined as the relative area of S_A covered with clouds at the observation time t. When determining B(t) from space photographs, one can have the shape and dimensions of S_A arbitrary. A change in S_A will lead to a change in B(t). As S_A increases, the probability distribution of B(t) is changing from a discrete to a continuous one that can be U or J-shaped. With further increase in S_A it becomes uniform, then bell-shaped and finally normal. For a discussion on that subject, see [50].

If S_A is very large such that B(t) has a normal distribution, the information on the local behavior of B within S_A is lost. In this paper the author presents a model for the distribution of B for a relatively small S_A. It is assumed that the number of states of cloudiness is finite: $B_1, B_2, \ldots, B_n$. Under various additional assumptions, the time variation of B(t) over S_A and space variation B(r) in passing from one S_A to another at a given time t in a zone of uniform cloudiness are modeled as discrete Markov processes. The space-time matrix of transition probabilities for states of cloudiness over any pairs of regions in the same uniform cloudiness zone separated by a distance r and time τ is denoted by

$$\Phi(\tau,r) = [\phi_{ij}(\tau,r)]_{n\times n} .$$

The assumptions that have been made imply that

$$\Phi(\tau,r) = Q(\tau)G(r)$$

where $Q(\tau) = [q_{ij}(\tau)]_{n\times n}$ is the matrix of transition probabilities of the states of cloud cover over a given averaging region that are separated by

τ units of time and $G(r) = [g_{ij}(r)]$ is the same matrix for different regions separated by a distance r.

The matrix $Q(\tau)$ is derived by solving the Kolmogorov equation

$$\frac{dQ}{d\tau} = QF \ , \quad Q(0) = 1_n \ ,$$

where $1_n = (1,1,\ldots,1)^T$ and $F = [f_{kj}]_{n\times n}$ is the matrix of transition intensities. For $k = 1,\ldots,n$ and $k \neq j$, $f_{kj} = 1/T_{kj} \geq 0$ and $f_{kk} = -\sum_{j=1}^{n} f_{kj}$, where T_{kj} is the average time during which the process remains in the state B_k before passing into B_j. T_{kj} is estimated from the data and the obtained matrix F is used as a first approximation for the derivation of $Q(\tau)$. The actual values of $Q(\tau)$ are obtained by solving an identification problem. Similar technique is applied to obtain $G(r)$.

After Q and G are derived, the author obtains the steady state distribution for $\Phi = QG$: $P_1, P_2, \ldots, P_n$ and derives $E[B] = \sum_{i=1}^{n} B_i P_i$.

In [68] the cloudiness values over a given surface point is modeled as a continuous Markov process of the diffusion type, ξ_z, characterized by two parameters: the transport coefficient a and diffusivity b, where z is the distance between the initial surface point and the current point under study. $0 \leq \xi_z \leq 1$, where 0 and 1 are reflection points. The author considers the random quantity $\theta(\xi_z)$ where $\theta(x)$ is a twice continuously differentiable function on [0,1]. If $u(x,y) = E[\theta(\xi_z) | \xi_0 = x]$, then $u(x,y)$ is the solution of the following partial differential equation:

$$a \ \frac{\nu u}{\nu x} + \frac{b}{2} \frac{\nu^2 u}{\nu x^2} = \frac{\nu u}{\nu z} \ ,$$

with the initial condition $u(x,0) = \theta(x)$ and boundary conditions $\frac{\nu u(0,z)}{\nu x} = \frac{\nu u(1,z)}{\nu x} = 0$. The solution $u(x,y)$ is obtained by using the Fourier method, and is a

function of the quantity $c = \frac{a}{b}$ and $\theta(x)$. By selecting appropriate functions $\theta(x)$, the author derives the cdf for cloudiness over the surface site, the transition probability of the cloudiness value at a distance z from initial point from segment $[\alpha_1,\alpha_2]$ to segment $[\beta_1,\beta_2]$ and the correlation function.

By comparing the theoretical formula for the cdf of cloudiness with the empirical cdf for cloudiness, one can obtain the value of $c = \frac{a}{b}$. The value of b is obtained by comparing the empirical and theoretical values of the correlation function. Thus the parameters a and b of the process can be determined from observed data.

The author goes on and uses the above approach to determine $E(\tau_x)$, where τ_x is the random distance along the ground track from the initial point where the cloudiness is equal to x, to the point where it reaches a threshold γ.

f. Regression Analysis and Inverse Problems ([3], [5], [11], [12], [32], [37], [39], [45], [46], [47], [50], [55], [57], [58], [60], [61], [62], [63], [64], [65], [69], [75]).

In [58], after several assumptions are made, the authors propose the following stochastic model of infrared radiation measurements in the present of stratified patchy clouds:

$$I(\nu) = (1 - \sum_{i=1}^{k} \alpha_i) I^{(c)}(\nu) + \sum_{i=1}^{k} \alpha_i I_i^{(\beta)}(\nu) \quad (1)$$

In (1), $I(\nu)$ is the outgoing radiation in the spectral range centered around the wave number ν, $I^{(c)}(\nu)$ and $I_i^{(\beta)}$ being the outgoing at the level p_{β_i} (at which the cloud layers are located), i = 1,...,k, in a cloudless atmosphere

and in a completely overcast sky, respectively, and $\underset{\sim}{\alpha} = (\alpha_1,\ldots,\alpha_k)^T$ is the k-dimensional vector of characteristics of multilayer clouds, each of whose components is uniformly distributed over [0,1]. Next, the authors proceed to express $I^c(\nu)$ and $I_i^{(\beta)}(\nu)$ by radiative transfer integral equations that relate the outgoing radiation to the pressure at the lower boundary of the corresponding cloud layer and the temperature as a function of that pressure. These integral equations are reduced to depend only on p_0, the pressure at ground level. The next transformation involves linearization of the resulting integral equations followed by finite dimensional approximation of the linearized equation.

The following model is obtained:

$$\underset{\sim}{A}\underset{\sim}{Q}(\underset{\sim}{\alpha})\underset{\sim}{x} + \underset{\sim}{B}\underset{\sim}{\alpha} + \underset{\sim}{\varepsilon} = \underset{\sim}{f} \tag{2}$$

where $E(\underset{\sim}{\varepsilon}) = \underset{\sim}{0}$, S is the sample estimate of $E(\underset{\sim}{\varepsilon}\underset{\sim}{\varepsilon}^T)$, $\underset{\sim}{Q}(\underset{\sim}{\alpha}) = (1 - \sum_{i=1}^{k} \alpha_i)E_m + \sum_{i=1}^{k} \alpha_i H_i$, $B = (b_1,\ldots,b_k)$, $\underset{\sim}{b}_i = A(H_i - E)\underset{\sim}{\bar{T}}$, A is an $m \times m$ matrix describing the finite dimensional approximation of linearized integral operators, E_m is an $m \times m$ identity matrix, H_i are $m \times m$ matrices, $\underset{\sim}{\bar{T}}$ is the vector of sample means, $x_j = T(p_j) - ET(p_j)$ and f_j is the deviation of the measured outgoing radiation by remote sensing $\underset{\sim}{I}_j$ from the calculated one I_j.

A satellite-borne detector collects data on the outgoing radiation at n spectral channels: $\underset{\sim}{I}(\nu_1),\ldots,\underset{\sim}{I}(\nu_n)$, in the present of additive independent random noice $\underset{\sim}{\varepsilon}$. Based on these data and model (2), the authors, using ideas from nonlinear regression rechniques, derive estimates for $\underset{\sim}{x}$ and $\underset{\sim}{\alpha}$. They introduce $\underset{\sim}{y} = \underset{\sim}{Q}\underset{\sim}{x}$, neglect its dependence on $\underset{\sim}{\alpha}$ and by assuming that $\underset{\sim}{y}$, $\underset{\sim}{x}$ and $\underset{\sim}{\alpha}$ are deterministic, derive the following explicit estimates for $\underset{\sim}{\alpha}$ and $\underset{\sim}{x}$:

$$\begin{aligned} \underset{\sim}{\hat{\alpha}}_I &= (B^T A A^T + B B^T)^{-1}\underset{\sim}{f} \\ \underset{\sim}{\hat{x}}_I &= \underset{\sim}{Q}^{-1}(\underset{\sim}{\hat{\alpha}}_I)(AA^T + BB^T)^{-1}\underset{\sim}{f} \,. \end{aligned} \tag{3}$$

Since $\hat{x}_I$ depends on f nonlinearly, it is impossible to derive explicit expressions for the covariance matrix of $\hat{x}_I$. The authors also claim that these estimates are not accurate. To improve the accuracy of these estimates the authors assume that x and α are random vectors whose first two sample moments are known $\bar{x}$, R_x, $\bar{\alpha}$ and R_α, where $E(x) = 0$. The best linear unbiased estimates of x and α are:

$$\begin{aligned} \hat{x}_{II} &= L_x[f - B\bar{\alpha}] \\ \hat{\alpha}_{II} &= \bar{\alpha} + L_\alpha[f - B\bar{\alpha}], \end{aligned} \tag{4}$$

where $L_x = R_x Q^T(\bar{\alpha}) A^T G^{-1}$, $L_\alpha = R_\alpha B^T G^{-1}$, $G = A\Theta A^T + BR_\alpha B^T + S$ and $\Theta = Q(\bar{\alpha}) R_x Q^T(\bar{\alpha})$. The covariance matrices for these estimates are estimated by:

$$\begin{aligned} D(\hat{x}_{II}) &= R_x - R_x Q^T(\bar{\alpha}) A^T G^{-1} A Q(\bar{\alpha}) R_x \\ D(\hat{\alpha}_{II}) &= R_\alpha - R_\alpha B^T G^{-1} B R_\alpha . \end{aligned} \tag{5}$$

Model (2) and estimates (4) and (5) are extended for other related problems.

In this article, [65], the authors present the following statistical model that relates the vector measurement of radiation characteristics y, with the vertical ozone profile in the atmosphere x:

$$y = A[x] + \varepsilon ,$$

where $y = (y_1, \ldots, y_s)^T$ ($s = mk$, where m is the number of spectral intervals and k is the number of space points or directions per interval), $x = (x_1, \ldots, x_n)^T$, $A[x]$ is a nonlinear function of x, and ε the random error. The components of ε represent the errors that occur in the measurements and in the model itself. We assume that $E(\varepsilon) = 0$ and Ω is the sample estimate of $E(\varepsilon\varepsilon^T)$. Moreover, we assume that we have estimated on the basis of a sample

$\bar{x}$, $\bar{y}$, $R = \frac{1}{n}\{[x - \bar{x}][x - \bar{x}]^T\}$, $Q = \frac{1}{s}\{[y - \bar{y}][y - \bar{y}]^T\}$ and

$K = \frac{1}{\sqrt{ns}}\{[y - \bar{y}][x - \bar{x}]^T\}$. Let $x = E(x) + \Delta x$ and assume that Δx and ε are independent.

Three methods, based on the least squares principle, for estimating x from y are discussed:

1. Formal Least Squares Method. It is assumed that (x,y) has a multivariate normal distribution. If the first two moments of the distribution are known, then the LSE has the form

$$\hat{x} = \bar{x} + K^T(Q + \Omega)^{-1}[y - \bar{y}] ,$$

and the covariance matrix of $\hat{x}$ is estimated by

$$\hat{R} = R - K^T(Q + \Omega)^{-1}K .$$

2. Stochastic Linearization Method. This approach assumes

$$y = b + C\Delta x ,$$

where $b = (b_1,\ldots,b_n)$ and $C = (c_{ij})$ are the unknown coefficients. The optimum choice of parameters is determined by:

$$b = y \quad \text{and} \quad C = KR^{-1} .$$

In this case the LSE of x is

$$\hat{x}_1 = \bar{x} + K^T(KR^{-1}K^T + \Omega)^{-1}[y - \bar{y}] ,$$

and the covariance matrix of $\hat{x}_1$ is estimated by

$$\hat{R}_1 = R - K^T(KR^{-1}K^T + \Omega)^{-1}K .$$

3. Analytic Linearization Method. In this case we approximate the functional relationship between Δy and Δx by

$$\Delta y = A'[x_0]\Delta x + \varepsilon,$$

where $A'[x_0]$ is the matrix of the partial derivatives at a certain point x_0

(sometimes $\underset{\sim}{x}_0 = E(\underset{\sim}{x})$). The LSE of x is

$$\hat{\underset{\sim}{x}}_2 = \bar{\underset{\sim}{x}} + R(A')^T[A'R(A')^T + \Omega]^{-1}[\underset{\sim}{y} - \bar{\underset{\sim}{y}}]$$

and the covariance matrix of $\hat{\underset{\sim}{x}}_2$ is estimated by

$$\hat{R}_2 = R - R(A')^T[A'R(A')^T + \Omega]^{-1}A'R \ .$$

To evaluate these techniques, the authors consider several information-content measures:

(a) the Shannon quantity of information of the solution component x_i that is present in the values of $\underset{\sim}{y}$, which is given by $I(x_i,\underset{\sim}{y}) = \log_2(r_{ii}/\hat{r}_{ii})$, and

(b) the multiple correlation coefficient of x_i and $\underset{\sim}{y}$ given by $d_i = 1 - (\hat{r}_{ii}/r_{ii})^2$, where r_{ii}^2 and $\hat{r}_{ii}^2$ are the diagonal elements of the matrices R and appropriate $\hat{R}$, respectively.

In [61] the authors are concerned with estimating the relative geopotential without solving the inverse problem for the vertical temperature distribution.

Let $\Delta\underset{\sim}{T} = \underset{\sim}{T} - E(\underset{\sim}{T})$, $\Delta\underset{\sim}{L} = \underset{\sim}{L} - E(\underset{\sim}{L})$ and $\Delta H^o(p)$ be the vectors of deviations from the temperature and radiation and the deviation of the relative geopotential of a surface with pressure p, respectively. After appropriate transformations, the following approximate algebraic model is presented:

(1) $\Delta\underset{\sim}{L} = A\Delta\underset{\sim}{T}$

(2) $\Delta H^o(p) = [\underset{\sim}{r}(p)]^T\Delta\underset{\sim}{T}$,

where $\Delta\underset{\sim}{T} = (\Delta T(p_n),\ldots,\Delta T(p_1))^T$, $\Delta\underset{\sim}{L} = (\Delta L(v_1),\ldots,\Delta L(v_m))^T$, A is an $m \times n$ matrix and $\underset{\sim}{r}(p)$ a vector of known constants.

The satellite observations are described by the model

$$\Delta\underset{\sim}{L} = A\Delta\underset{\sim}{T} + \underset{\sim}{\varepsilon} \ ,$$

where $\underset{\sim}{\varepsilon}$ is the error of measurements and the model used. The authors assume that $\underset{\sim}{\varepsilon}$ has a multivariate normal distribution with mean $\underset{\sim}{0}$ and a known covariance matrix $K^2_{\underset{\sim}{\varepsilon}}$. The best linear unbiased estimate of ΔH^o is given by:

$$\widehat{\Delta H^o} = \hat{\underset{\sim}{q}}^T \Delta \underset{\sim}{L} ,$$

where $\hat{\underset{\sim}{q}} = (I - \bar{K}_{\underset{\sim}{\varepsilon}} \bar{K}_{\underset{\sim}{\varepsilon}})(A^-)^T \underset{\sim}{r}$, $\Delta \underset{\sim}{L} = \underset{\sim}{L} - \bar{\underset{\sim}{L}}$, and $K_{\underset{\sim}{\varepsilon}}$ is the positive square root of $K^2_{\underset{\sim}{\varepsilon}}$, $\bar{K}_{\underset{\sim}{\varepsilon}} = K_{\underset{\sim}{\varepsilon}}(I - AA^-)$ and $-$ is the notation for pseudo inverse operation. The above estimate coincides with the ordinary least squares estimate (OLSE) for which $\tilde{\underset{\sim}{q}} = (A^-)^T \underset{\sim}{r}$ if and only if $\bar{K}_{\underset{\sim}{\varepsilon}}^- \bar{K}_{\underset{\sim}{\varepsilon}} = \bar{K}_{\underset{\sim}{\varepsilon}}^- K_{\underset{\sim}{\varepsilon}}$. The authors assume in their analysis that $K^2_{\underset{\sim}{\varepsilon}} = \sigma^2 I$ which guarantees the OLSE $\tilde{\underset{\sim}{q}}$.

The general solution to (1) is:

$$\Delta \bar{\underset{\sim}{T}} = A^- \Delta \bar{\underset{\sim}{L}} + \underset{\sim}{s} ,$$

where $\Delta \bar{\underset{\sim}{T}} = \underset{\sim}{T} - \bar{\underset{\sim}{T}}$, $\Delta \bar{\underset{\sim}{L}} = \underset{\sim}{L} - \bar{\underset{\sim}{L}}$, $\underset{\sim}{s}$ is any vector in the null space of A. Similarly, one obtains

$$\Delta H^o - \widehat{\Delta H^o} = \underset{\sim}{r}^T P \Delta \underset{\sim}{T} - \underset{\sim}{r}^T A^- \underset{\sim}{\varepsilon} ,$$

where $\underset{\sim}{s} = P \Delta \bar{\underset{\sim}{T}}$ and $P = I - A^- A$. Moreover,

$$\hat{\sigma}^2(\Delta H^o) = \underset{\sim}{r}^T P K^2_{\underset{\sim}{T}} P \underset{\sim}{r} + \underset{\sim}{r}^T A^- K^2_{\underset{\sim}{\varepsilon}} (A^-)^T \underset{\sim}{r} ,$$

where $K^2_{\underset{\sim}{T}}$ is the covariance matrix of $\Delta \underset{\sim}{T}$. If $K^2_{\underset{\sim}{\varepsilon}} = \sigma^2 I$, then the above formula reduces to

$$\hat{\sigma}^2(\Delta H^o) = \underset{\sim}{r}^T P K^2_{\underset{\sim}{T}} P \underset{\sim}{r} + \sigma^2 \underset{\sim}{r}^T (A^T A)^- \underset{\sim}{r} .$$

The authors proceed to investigate the problem of selecting the spectral intervals that will minimize $\hat{\sigma}^2(\Delta H^o)$.

In [64] The authors assume that the input information for analysis of the random temperature field $\underset{\sim}{x}$ consists not only of aerological data, but also of remotely sensed data. For some coordinates $\underset{\sim}{z}_1, \underset{\sim}{z}_2, \ldots, \underset{\sim}{z}_n$ on Earth

surface, the deviations of $\underset{\sim}{x}(\underset{\sim}{z}_i)$ from their means $E[\underset{\sim}{x}(\underset{\sim}{z}_i)]$ are related to the variations of the measured quantities (some aerological and some remotely sensed) $\underset{\sim}{Y}(\underset{\sim}{z}_i)$ from their means $E[\underset{\sim}{Y}(\underset{\sim}{z}_i)]$ via the statistical model:

$$(1) \qquad \underset{\sim}{Y}(\underset{\sim}{z}) - E[\underset{\sim}{Y}(\underset{\sim}{z})] = \underset{\sim}{A}(\underset{\sim}{z})\{\underset{\sim}{x}(\underset{\sim}{z}) - E[\underset{\sim}{x}(\underset{\sim}{z})]\} + \underset{\sim}{\varepsilon}.$$

In (1) $\underset{\sim}{A}(\underset{\sim}{z})$ is the matrix that approximates the integral operator of the transfer equations and $\underset{\sim}{\varepsilon}(\underset{\sim}{z})$ is a random error. Given the observed data at $\underset{\sim}{z}_1,\ldots,\underset{\sim}{z}_n$ the authors are interested in estimating the meteorological variable $\underset{\sim}{x}$ at some point $\underset{\sim}{z} = \underset{\sim}{z}_0$. The authors point out that the matrix A can be ill conditioned which presents difficulties for estimating $\underset{\sim}{x}(\underset{\sim}{z}_0)$. They assume that the sample mean $\overline{\underset{\sim}{x}}(\underset{\sim}{z}_i)$ and the sample covariance matrix of $\underset{\sim}{x}(\underset{\sim}{z}_i)$, $R(\underset{\sim}{z}_i)$, and the sample cross covariance matrix, $R(\underset{\sim}{z}_i,\underset{\sim}{z}_j)$, have been obtained from an a priori data. Moreover, $E[\underset{\sim}{\varepsilon}(\underset{\sim}{z})] = 0$, $E[\underset{\sim}{\varepsilon}(\underset{\sim}{z})\underset{\sim}{\varepsilon}(\underset{\sim}{z}')^T] = 0$ for $\underset{\sim}{z} \neq \underset{\sim}{z}'$, and $\underset{\sim}{R}_{\varepsilon}$ is the sample estimate of $E[\underset{\sim}{\varepsilon}(\underset{\sim}{z})\underset{\sim}{\varepsilon}(\underset{\sim}{z})^T]$. Hence $\overline{\underset{\sim}{Y}}(\underset{\sim}{z}) = \underset{\sim}{A}(\underset{\sim}{z})\overline{\underset{\sim}{x}}(\underset{\sim}{z})$ and $\underset{\sim}{R}_Y(\underset{\sim}{z}) = \underset{\sim}{A}(\underset{\sim}{z})\underset{\sim}{R}(\underset{\sim}{z})\underset{\sim}{A}^T(\underset{\sim}{z}) + \underset{\sim}{R}_{\varepsilon}(\underset{\sim}{z})$. The estimate of $x(z_0)$ that minimizes the mean square error is given by:

$$\hat{\underset{\sim}{x}}(\underset{\sim}{z}_0) = \overline{\underset{\sim}{x}}(\underset{\sim}{z}_0) + \underset{\sim}{R}_{\underset{\sim}{x}\underset{\sim}{Y}}\underset{\sim}{R}_{\underset{\sim}{Y}}^{-1}(\underset{\sim}{Y} - \overline{\underset{\sim}{Y}}) ,$$

where

$$\{\underset{\sim}{R}_{\underset{\sim}{Y}}\}_{ij} = \underset{\sim}{A}(\underset{\sim}{z}_i)\underset{\sim}{R}(\underset{\sim}{z}_i,\underset{\sim}{z}_j)\underset{\sim}{A}^T(\underset{\sim}{z}_j) + \delta_{ij}\underset{\sim}{R}_{\varepsilon}(\underset{\sim}{z}_i) ,$$

and

$$\underset{\sim}{R}_{\underset{\sim}{x}\underset{\sim}{Y}} = [R(\underset{\sim}{z}_0,\underset{\sim}{z}_1)\underset{\sim}{A}^T(\underset{\sim}{z}_1),\ldots,R(\underset{\sim}{z}_0,\underset{\sim}{z}_n)\underset{\sim}{A}^T(\underset{\sim}{z}_n)]$$

and $\delta_{ij} = 1$ if $i = j$ and 0 otherwise. The covariance matrix of $\hat{\underset{\sim}{x}}(\underset{\sim}{z}_0)$ is estimated by

$$\hat{\underset{\sim}{R}}(\underset{\sim}{z}_0) = \underset{\sim}{R}(\underset{\sim}{z}_0) - \underset{\sim}{R}_{\underset{\sim}{x}\underset{\sim}{Y}}\underset{\sim}{R}_{\underset{\sim}{Y}}^{-1}\underset{\sim}{R}_{\underset{\sim}{x}\underset{\sim}{Y}}^T .$$

The authors state that the inversion of $R_{\underset{\sim}{Y}}$ is quite difficult because n is large and they use a recursive algorithm to overcome this difficulty.

The size of the sample, n, is called a filter of order n and the estimation procedure, $\hat{\underset{\sim}{x}}(\underset{\sim}{z}_0) = \hat{\underset{\sim}{x}}(\underset{\sim}{z}_0|\underset{\sim}{z}_1,\ldots,\underset{\sim}{z}_n)$, if $\underset{\sim}{z}_0 \neq \underset{\sim}{z}_j$, $j = 1,\ldots,n$, is called optimum extrapolation if $n = 1$ and optimum interpolation if $n > 1$. When all n observations are taken successively at $\underset{\sim}{z}_0$ (i.e., $\underset{\sim}{z}_j = \underset{\sim}{z}_0$, $j = 1,\ldots,n$), then the estimation procedure is called spatial adjustment (filtration). The performance of these procedures are evaluated in the paper. The authors have studied how the size of the filter and the actual location of the points $\underset{\sim}{z}_1,\ldots,\underset{\sim}{z}_n$ in relation to $\underset{\sim}{z}_0$ affect the accuracy of the estimation procedure. Also investigated was the effect of combining the aerological data with remotely sensed data in the estimation procedure.

g. <u>Spatial Analysis</u> ([20], [22], [34], [35], [36], [45], [46], [48], [61], [62], [64], [72], [76], [77]).

In [34] the author describes the three-dimensional structure of the ocean-atmosphere radiance field that has been obtained by measuring reflected solar radiation in the visible range (.4 - .7 μm) at two heights (.3 and 10 km) in the atmosphere.

The author calculates the spatial radiance profiles $I_i^{\uparrow}(\lambda_k)$, $i = 1,2,\ldots,N$, in individual spectral bands of .01 μm in the .4 - .7 μm range. From these calculations it is deduced that the radiance of the ocean-atmosphere system is a random function of the spatial coordinates and time. The variability of the atmospheric optical parameters is quite high. The effect of the

atmosphere on $I^{\uparrow}(\lambda_k)$ is most noticeable at the height of 10 km., where the maximum spread of the spectra for some values of λ_k reaches 75%. There is a strong correlation of large radiance variations at each of the measurement heights for different wavelengths λ_k.

The author assumes that the spatial structure of the radiance field of the ocean surface or the ocean-atmosphere system is stationary and therefore is characterized by the autocorrelation function $R_{I_\lambda}(\rho)$. These functions are plotted and examined for the two heights and several wavelengths. By comparing the autocorrelation function at the beginning, middle, and end of the .4 - .7 μm band, the author studies the variation over the spectrum of contributions to the radiance field.

The spectral densities, $S_{I_\lambda}(\omega)$, are calculated as well and are used in the investigation of the nonuniformities in the spatial structure of the radiance field.

In this article, [77], the authors simulated Goudrian's theoretical model for plant canopy. The plant canopy is modeled by a horizontally infinite layer whose bioelements have a Poisson spatial distribution. The layer consists of m sublayers L_s thick each. There is no mutual shadowing of bioelements within these sublayers. The azimuthal distribution of bioelements is assumed to be uniform, hence their orientations are defined by their inclinations Θ_b. Functions $f(\Theta_b)$ are defined for nine 10° sectors in the range $0 - 90^\circ$, such that $\sum_{\Theta_b=1}^{9} f(\Theta_b) = 1$. The average projection of the entire layer L onto the direction along the zenith angle Θ_s is

$$E[G(\Theta_s)] = \sum_{\Theta_b=1}^{9} f(\Theta_b) G(\Theta_s, \Theta_b) ,$$

where $G(\Theta_s,\Theta_b)$ is the azimuth-averaged projected leaf surface inclined by Θ_b into the Θ_s direction. The contribution at each 10^o sector to the total irradiance of the horizontal surface is given by a symmetric weight function $B_U(\Theta_s)$. The fraction of the leaf surface area that interacts with the radiation within the L_s layer is

$$M_i(\Theta_s) = L_s E[G(\Theta_s)]/\sin(\Theta_s) .$$

Based on these quantities and several other assumptions, the Goudrian model specifies the fraction of radiation attenuated by the jth layer.

The authors state that the assumption of a random spatial distribution of the bioelements restricts the application of Goudrian's model. The model is reasonable for vegetation canopies that have no well ordered structure, like crops and meadows. The authors compared measured and predicted (from the simulation) reflectances for winter wheat fields. The results support the applicability of Goudrian's model.

In the simulation of Goudrian's model to study the reflectance b_λ, the effect of the following two parameters was examined in detail.

(1) Leaf area index, that characterizes soil coverage by bioelements and is related to percentage of vegetative cover.

(2) Leaf orientation.

h. <u>Spectral Analysis</u> ([5], [20], [22], [33], [34], [38], [57], [59], [63], [72], [76]).

In [48] the authors analyze the fluctuations of the radiative components of the ocean-atmosphere system and the ocean surface with respect to monthly averages of the radiation budget.

The radiation budget of the Earth's surface is determined from the following transfer equation:

$$\begin{Bmatrix} I(z,\mu) \\ \tilde{I}(z,\mu) \end{Bmatrix} = \frac{1}{2\pi} \sum_{n=0}^{2n+1} \begin{Bmatrix} \psi_n(z) \\ \tilde{\psi}_n(z) \end{Bmatrix} P_n(\mu) ,$$

where $I(z,\mu)$ and $\tilde{I}(z,\mu)$ are fields of the shortwave and longwave radiation, z being the vertical coordinate, μ the cosine of the zenith angle, $(\psi_n, \tilde{\psi}_n)$ are vectors of the intensity angular distribution moments and P_n are Legendre polynomials. It follows that the ground radiation budget B_g is given by:

$$B_g = \psi_1(z = 0) + \tilde{\psi}_1(z = 0) .$$

The values of ψ_1 and $\tilde{\psi}_1$ are tabulated for various models of the atmosphere. The effect of the particular models in determining B_g from satellite data is very small because of the process of averaging the data over time and experimental conditions.

The authors consider year-to-year variations of the two radiative components in an analysis of measurements in North Atlantic for the period of 45 months in 1974-1978. Their measure of variation is the variances of the entire 45 month data with respect to the monthly averages within 2.5°-lattitude by 2.5°-longitude grid. Figures with isolines for the monthly averages of the radiation budget are presented and carefully analyzed. Zones of significant year-to-year variations of radiation budget were identified.

To study the spatial distribution of the effect of zones of significant year-to-year radiation, the authors employ a two-dimensional Fourier analysis of the fields of the radiation budget and its components. Among other things, the authors estimate the spectral density and the correlation function of the

anomalies of the two-dimensional radiation budget field. By analyzing the spectral density function, the authors explain the structure of the spatial variability (two-dimensional variance). Principal component analysis is performed on the correlation matrices and the radiation budget anomalies are explained in terms of a few principal components.

The following topics were not discussed in this section because of their elementary nature: Analysis of Variance ([30]), Optimal Design ([26], [53]) and Principal Component Analysis ([1], [28]).

4. SUMMARY

In conclusion of this survey article, I would like to stress again the fact that the following three journals give a pretty complete account of the statistical techniques used by Russian scientists in remote sensing: Soviet Journal of Remote Sensing, Izvestiya, Atmospheric and Oceanic Physics, and Soviet Meteorology and Hydrology.

It becomes clear from this survey that almost all the areas of statistics have been used in the analysis of remotely sensed data. There are quite a few statistical techniques that have not been used, especially those that represent the advances in the last ten to fifteen years. For example, no modern exploratory data analysis techniques (graphics, outlier detection, regression diagnostics, etc.), robust statistical methods or sample reuse techniques (bootstrap, jackknife) have been used at all. On the other hand, the mathematical models that govern the observed processes are quite complex, as are the instrumentation that is used in collecting the data. That is why the author of this article believes that there is a need for collaboration between the statisticians and the scientists that study remotely sensed data.

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In this article we review the major areas of remote sensing in the Soviet literature that use statistical methods to analyze observed data. For each of the areas, we list the problems that have been studied and briefly describe the statistical techniques that have been used.

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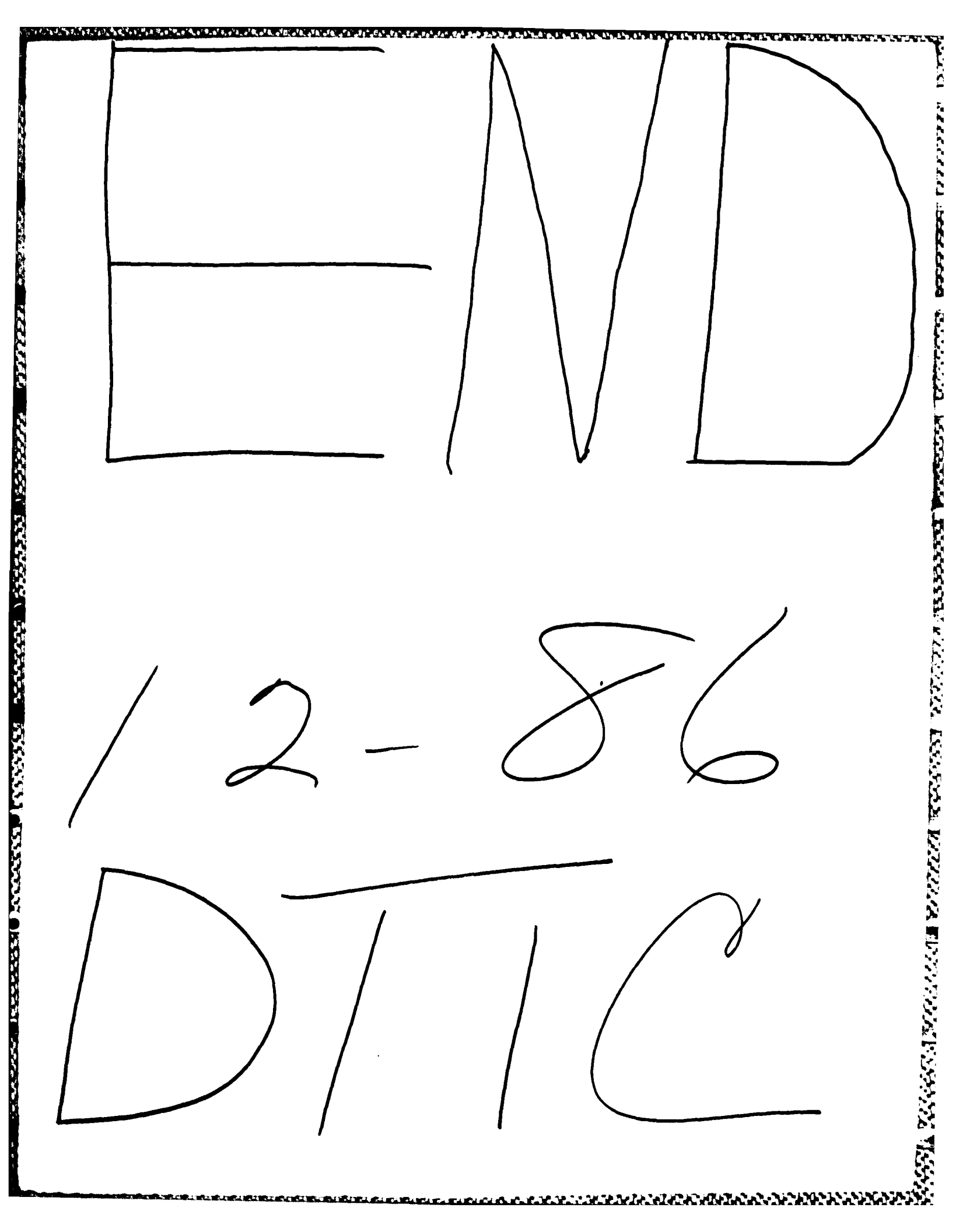
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